THE ABSTRACTS OF THE TALKS

2022 ALGEBRA AND BEYOND

A CONFERENCE IN HONOR OF THE MATHEMATICAL CONTRIBUTIONS OF
MICHAEL J. LARSEN
JUNE 3 - 5, 2022, INDIANA UNIVERSITY

1. The Schedule

The invited lectures and Professional Development Panel are in **Rawles 100**.
The banquet is in the **Frangipani room** of the **Indiana Memorial Union.**

all times EST	June 3 (Fri.)	June 4 (Sat.)	June 5 (Sun.)
8:45 - 9:30	Registration/coffee		
9:30 - 10:30	Chun Yin Hui	Carlos Simpson	Ayelet Lindenstrauss
10:40 - 11:40	Melanie Wood	Shekhar Khare	Ching-Li Chai
11:40 - 13:15	Lunch	Lunch	
13:15 - 14:15	Aner Shalev	Renee Bell	
14:20 - 15:20	Robert Guralnick	Donna Testerman	
15:20 - 15:50	Coffee Break	Coffee Break	
15:50 - 17:15	Contributed sessions #1	Contributed sessions #2	
17:20 - 18:20	Bo-Hae Im	Professional Development Panel	
18:30 - 22:00		Banquet	

Contributed session #1:
Swain East 105: Eilidh McKemmie, Itay Glazer
Swain East 140: Joshua Ruiter, Chris Schroeder, Sung Min Le

Contributed session #2:
Swain East 105: Erik Wallace, Shiang Tang
Swain East 140: Hung Nguyen, Be'eri Greenfield

2. Abstracts of invited talks

1. **Speaker:** Renee Bell (University of Pennsylvania)
   Title: Monodromy of Tamely Ramified Covers of Curves
   Abstract: For a curve X over an algebraically closed field of characteristic 0, the etale fundamental group π_1^{et} can be described solely in topological terms, but in characteristic p, dramatic differences and new phenomena have inspired many conjectures, including analogues of the inverse Galois problem. Let k be an algebraically closed field of characteristic p and let X be the

projective line over k with three points removed. A theorem of Harbater and Raynaud characterizes the finite quotients of $\pi_1^{et}(X)$, but it is not known which of those finite quotients arise as Galois groups of *tamely ramified* covers of the projective line. In joint work with Booher, Chen, and Liu, we show that for each prime $p \geq 5$, there are families of tamely ramified covers with monodromy the symmetric group S_n or alternating group A_n for infinitely many n, producing these covers from moduli spaces of elliptic curves, and relating the fiber of these covers to the Markoff surface.

2. **Speaker:** Ching-Li Chai (University of Pennsylvania)
Title: Orbital rigidity and Tate-linear formal varieties
Abstract: A Tate-linear formal varieties is a smooth formal schemes over a field k of characteristic $p > 0$ which is assembled from a finite number of isoclinic p-divisible groups over k under the guidence of a projective family of finite unipotent group schemes over k. These formal varieties are motivated by joint work with Frans Oort on the local structure of central leaves on moduli spaces of abelian varieties in characteristic p.

   This talk is focused on the orbital rigidity property of Tate-linear formal varieties. We will begin with an example of this rigidity phenomenon, when the Tate-linear formal variety is a formal torus, or more generally a p-divisible formal group. We will explain the new ingredients which are needed to reach the simplest case beyong p-divisible groups, namely bi-extensions of p-divisible formal groups. Tate-linear formal varieties will be introduced as the concept needed in the formulation of the general rigidity result: Suppose that a compact p-adic Lie group G operates on a Tate-linear formal variety V such that the induced action on every Jordan-Holder component of V is infinite. Then every reduced irreducible formal subvariety of V which is stable under G is a Tate-linear formal variety.

3. **Speaker:** Robert Guralnick (USC)-Zoom
Title: Strongly Dense Subgroups and Generation of Simple Groups
Abstract: A strongly dense subgroup of a semisimple algebraic group is one in which every nonabelian subgroup is Zariski dense. The existence of these was proved by Breuillard, Greene, Guralnick and Tao and there is an improvement of these results by Breuillard, Guralnick and Larsen. We will discuss the proofs of these results and applications to various problems.

4. **Speaker:** Chun Yin Hui (University of Hong Kong)-Zoom
Title: Big images and irreducibility of Galois representations
Abstract: Let K be a number field and E/K be an elliptic curve with no complex multiplication over K (i.e., $\mathrm{End}_K(E) = \mathbb{Z}$). J.-P. Serre proved that the ℓ-adic representation $\rho_\ell : \mathrm{Gal}_K \to \mathrm{GL}_2(\mathbb{Q}_\ell)$ attached to E/K is irreducible for all ℓ and residually irreducible for almost all ℓ. Moreover, if E has no complex multiplication over $\overline{K}$ (i.e., $\mathrm{End}_{\overline{K}}(E) = \mathbb{Z}$) then the residual image of ρ_ℓ is $\mathrm{GL}_2(\mathbb{F}_\ell)$ for almost all ℓ. A generalization of Serre's big images results is this. Let $\{\rho_\ell : \mathrm{Gal}_K \to \mathrm{GL}_n(\mathbb{Q}_\ell)\}_\ell$ be a compatible system of semisimple ℓ-adic representations satisfying mild conditions. For almost all ℓ, if V_ℓ is an irreducible subrepresentation of $\rho_\ell \otimes \overline{\mathbb{Q}}_\ell$ of type A (e.g., when $\dim V_\ell \leq 3$) then V_ℓ is residually irreducible. We describe the ideas and tools behind this result and give an application to the irreducibility of some automorphic Galois representations.

5. **Speaker:** Bo-Hae Im (Korea Advanced Institute of Science and Technology, Korea)
Title: Rank of elliptic curves over large field extensions

Abstract: I will talk about the historical progress on my Ph.D. thesis problem which I worked under the supervision of Professor Michael Larsen. Especially, we consider when the ranks of elliptic curves over certain infinite extensions of number fields (or certain types of large extensions) are infinite, and also we consider them for abelian varieties. We present the partial results on this problem, which have been obtained by applying various methods. We introduce the definition of anti-Mordell-Weil fields (AMW) and conjectures and known results on AMW fields and ample fields.

6. **Speaker:** Chandrashekhar Khare (UCLA)
Title: Computational aspects of the Langlands program
Abstract: I will talk about various questions related to computing with modular forms and Galois representations. For instance one question is if there are around $p^3/48$ mod p Hecke eigenforms of level 1 as you vary the prime p. This is known to be an upper bound, and the best known lower bound is $p^2/8$. It would be of interest to get a lower bound of the order of $p^{2+\delta}$ for any $\delta > 0$.

7. **Speaker:** Ayelet Lindenstrauss (Indiana University)
Title: The Small Subcomplex of the Hochschild Complex
Abstract: The Hochschild homology of a k- algebra R (where k is a commutative ring) is the homology of a chain complex functorial in R whose rank, when that makes sense, grows exponentially with degree. It is of interest partly because of the Dennis trace map, which connects the algebraic K-theory of a ring to its Hochschild homology.

   If R is flat over k, the Hochschild homology $\mathrm{HH}^k_*(R)$ is isomorphic to $\mathrm{Tor}_*^{R\otimes_k R^{\mathrm{op}}}(R,R)$, which allows potentially smaller complexes to be used. In particular, for $R = k[x]/(f[x])$ with f a monic polynomial, there is a much smaller periodic complex. In our first paper together, Michael and I embedded this smaller complex in the standard one, connecting the easy-to-work-with version to the functorial one. This has been something I have used repeatedly in my career to calculate variants of Hochschild homology closer to algebraic K-theory, specifically cyclic homology and topological Hochschild homology, and I will discuss some of these calculations.

8. **Speaker:** Aner Shalev (Hebrew Univeristy)
Title: Characteristic covering numbers of groups and tensor product growth of representations
Abstract: I will discuss two recent joint works with Michael Larsen and Pham Tiep.

   The first studies words which are not an identity of any (non-abelian) finite simple group, and shows that short products of such words (in disjoint sets of variables) are surjective on ALL finite simple groups. This is carried out in a more general framework of characteristic collections of groups, and may be regarded as a generalization of work by Guralnick, Liebeck, O'Brien, Tiep and me (2018), which in turn extends the Odd Order Theorem.

   The second part of the talk will focus on representations of finite simple groups of Lie type, and of compact semisimple Lie groups. It is motivated by the theory of approximate subgroups, which establishes 3-step growth phenomena for simple groups of Lie type of bounded rank. We replace products of subsets by tensor products of representations, and establish some stronger 2-step growth results also in unbounded rank.

9. **Speaker:** Carlos Simpson (Université Cte d'Azur)
Title: Perspectives for computation in theoretical mathematics

Abstract: We'll look at some ways that the use of computers can lead to theorems in combinatorics, algebra, logic and related fields.

10. **Speaker:** Donna Testerman (EPFL, Switzerland)
Title: Reductive overgroups of regular unipotent elements in simple linear algebraic groups
Abstract: Each simple linear algebraic group defined over an algebraically closed field has a unique conjugacy class of elements (the so-called regular unipotent elements) which is dense in the variety of unipotent elements. This class plays an important role in the theory of these groups.

In this talk, we first trace some of the history of the study of closed subgroups containing regular unipotent elements, concentrating on SL_2-type subgroups, both finite and infinite. We describe some interesting applications of these rank one subgroups to the solution of a variety of problems in group and representation theory.

At the end of the talk, we will discuss a recent result with Gunter Malle (and related work of Bate, Martin and Röhrle) describing a certain property of disconnected positive-dimensional subgroups intersecting nontrivially the class of regular unipotent elements.

11. **Speaker:** Melanie Wood (Harvard University)
Title: The moment problem for random groups
Abstract: Distributions of real numbers are often recognized and characterized by their moments, the averages of X^k for each k over the distribution. Many times we are interested in distributions of algebraic structures, such as class groups of random number fields, or Selmer groups of random elliptic curves, or even non-abelian groups such as Galois groups of maximal unramified extensions of number fields, or fundamental groups of 3-manifolds. We will discuss results on the moment problem for random groups that allow us to recognize distributions such as these from their moments. This talk includes joint work with Weitong Wang and Will Sawin.

3. Abstracts of contributed talks

1. **Speaker:** Itay Glazer (Northwestern University)
Title: On singularity properties of word maps and applications to random walks on compact p-adic groups.
Abstract: To a word w in a free group F_r on a set of r elements, and a group G, one can associate a word map $w : G^r \to G$. When G is compact, such a word map induces a natural probability measure on G, and one can study the corresponding random walk. We consider the collection of random walks on $SL_n(\mathbb{Z}/p^k\mathbb{Z})$ induced by w, as p, k and n vary. It turns out that various mixing properties of these random walks have equivalent algebro-geometric characterizations in terms of the singularity properties of the word maps $w : SL_n(\mathbb{C})^r \to SL_n(\mathbb{C})$, and their concatenations (also called convolutions) $w^*w^* \cdots^* w : SL_n(\mathbb{C})^{rt} \to SL_n(\mathbb{C})$. We show that word maps on semisimple Lie groups and Lie algebras have nice singularity properties after sufficiently many self-convolutions (with bounds depending only on the word). As a consequence, we obtain some uniform results on the above collection of random walks. Based on a joint work with Yotam Hendel, see arXiv:1912.12556."

2. **Speaker:** Be'eri Greenfield (UCSD)
Title: Growth of unbounded subsets in nilpotent groups and random mapping statistics **Abstract:** Let G be an infinite group. Let $g(k, n)$ be the maximum number of length-n words over

an arbitrary k-letter subset of G. How does $g(k,n)$ behave? Obviously, $g(k,n)$ is at most k^n, and Semple-Shalev proved that if G is finitely generated and residually finite then $g(k,n) < k^n$ if and only if G is virtually nilpotent. It is then natural to ask how far $g(k,n)$ can get from k^n; for k fixed and n tending to infinity, $g(k,n)$ is polynomially bounded.

We quantify the Semple-Shalev Theorem at the other extreme, where $k = \Theta(n)$. Specifically, for a finitely generated residually finite group G, the ratio $g(k,n)/k^n$ either tends to zero (if and only if G is virtually abelian), or is greater than or equal to an explicitly calculated threshold. This limit threshold is derived from a Poisson distribution which depends on the asymptotics of k/n and cannot be improved: it is attained for (any) Heisenberg group. For higher free nilpotent groups, this ratio tends to 1.

Along the way, we find the probability that a random function $f : [n] \to [n]$ can be recovered from a suitable 'inversion set', and geometrically interpret our results via random paths in Z^n and the areas of their projected polygons. Finally, we provide a model-theoretic characterization of suboptimality of $g(k,n)$ by means of free sub-models and polynomial identities, which enables to generalize the discussion to various other classes of algebraic structures.

This is a joint work with Hagai Lavner.

3. **Speaker:** Eilidh McKemmie (Rutgers University)
Title: The probability of generating invariably a finite simple group
Abstract: We say a group is invariably generated by a subset if every tuple in the product of conjugacy classes of elements in that subset is a generating tuple. We discuss the history of computational Galois theory and probabilistic generation problems to motivate some results about the probability of generating invariably a finite simple group, joint work with Daniele Garzoni. We also highlight some methods for studying probabilistic invariable generation."

4. **Speaker:** Sung Min Lee (University of Illinois at Chicago))
Title: On the acyclicity of reductions of elliptic curves modulo primes in arithmetic progressions (joint work with Nathan Jones). **Abstract:** Let E be an elliptic curve defined over $\mathbb{Q}$. We consider the question of which arithmetic progressions have the property that, for all but finitely many primes in the arithmetic progression the elliptic curve reduction modulo p is NOT cyclic. In particular, we answer in the negative a question of Akbal and Glolu about the conditions on an integer pair (a,n) relative to E which guarantee that this property hold.

5. **Speaker:** Hung Nguyen (U. of Akron)
Title: Character codegrees of finite simple groups **Abstract:** The codegree of a character χ of a finite group G is $cod(\chi) := |G : Ker(\chi)|/\chi(1)$. This notion was first introduced and studied in the works of Chillag, Herzog, Mann, and Manz in the early 1990s, and has been recently proved to have remarkable connections with structure of finite groups. I will discuss a work in progress where we aim to show that finite simple groups have very distinctive character codegrees. For instance we show that a finite group G has the same codegrees counting multiplicity with a finite simple group H if and only if G is isomorphic to H. The proofs make essential use of some deep results on character degrees of finite simple groups, including the Malle-Larsen-Tiep bound for the largest character degree of finite groups of Lie type.

6. **Speaker:** Joshua Ruiter (Michigan State University)
Title: Abstract homomorphisms of special unitary groups
Abstract: A longstanding conjecture of Borel and Tits predicts that an abstract homomorphism of between the groups of rational points of algebraic groups can be written as a composition of an algebraic morphism and something induced by a morphism of algebras. In recent work with Igor Rapinchuk, I extend known methods for split groups in resolving the BT conjecture to a class of quasi-split special unitary groups.

7. **Speaker:** Chris Schroeder (Binghamton University)
Title: The 2-parts of relative character degrees.
Abstract: Let G be a finite group. The following result, proved by Higgs and Moretó, was used by Navarro and Tiep in their proof of Brauer's height zero conjecture for 2-blocks of maximal defect: suppose Z is a normal subgroup of G and θ is a G-invariant irreducible character of Z. If the relative degree $\chi(1)/\theta(1)$ is odd for every irreducible character χ of G lying over θ, then G/Z is solvable. The original proof of this result used the language of projective representations. We give a character theoretic proof and report progress on a conjecture extending this result to the case where the relative degrees are not divisible by 4. Our work extends to the projective setting a result of Lewis that if G is nonsolvable and $4 \nmid \chi(1)$ for all irreducible characters χ of G, then $G \cong A_7 \times S$ with S solvable.

8. **Speaker:** Shiang Tang (Purdue University)
Title: Lifting local Galois representations in the $\ell \neq p$ case (joint with J. Booher)
Abstract: given an ℓ-adic field F and a connected split reductive group G, it is natural to ask if any G-valued continuous mod p representation of the absolute Galois group of F has a characteristic zero lift. For G equals the general linear groups, symplectic groups and orthogonal groups, this question is answered in the affirmative by the work of Clozel-Harris-Taylor and Booher. Their arguments are built upon basic representation-theoretic tools such as tensor product and induction. For general G, new ideas are needed since no analogs of these seem to be known. We prove new instances of the above problem by introducing the notion of "decomposition types", which are certain non-connected subgroup schemes of G. For GL_n, they capture the multiplicities of irreducible summands of a completely reducible representation, and for general G, they appear to be a suitable remedy for the lack of such notions.

9. **Speaker:** Erik Wallace (Hartwick University)
Title: Semi-magic squares with symmetries arising from Galois representations on Elliptic Curves over $\mathbb{Q}$ or $\mathbb{F}_p$. **Abstract:** In an article from 2010 Edray Goins introduced the concept of semi-magic squares with points on elliptic curves. Here we consider elliptic curves defined over $\mathbb{Q}$ or $\mathbb{F}_p$, and use torsion-points defined over an algebraic extension to construct semi-magic squares on which the $Gal(\bar{\mathbb{Q}}/\mathbb{Q})$ or $Gal(\bar{\mathbb{F}}_p/\mathbb{F}_p)$ can act. There are several ways of identifying an ordinary semi-magic square with a semi-magic square with points on elliptic curves. For one of these, we describe conditions for invariance under the Galois representation in terms of the torsion that occurs over the base fied. For another, we show the existence of semi-magic squares that remain invariant under the split-cartan action in any basis."